

# Model of Magnetostriction and Magnetization for Galfenol Rods with Considering the Effects of Anisotropy and Dynamic Losses

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The model of magnetostriction and magnetization for Galfenol is established with considering the anisotropy and dynamic losses, based on the theory of free energy, Jiles-Atherton model and Stoner-Wohlfarth model. The magnetostriction ( $\lambda$ - $H$ ) and magnetization ( $M$ - $H$ ) for Galfenol rods under different magnetic fields and stresses have been calculated by the constitutive model. The measurement of magnetic properties for Galfenol rod specimen is made by the WK-22 magnetic parameter measurement system. The calculated and experimental results match very well. It means that the proposed model can effectively describe the effects of anisotropy, stress and dynamic losses on the magnetostriction and magnetization of Galfenol rods.

*Index Terms*—Anisotropy, energy loss, magnetization, Galfenol.

## I. INTRODUCTION

Galfenol magnetostrictive material has the characteristics of fast response speed, giant ductility and low saturation magnetic field. These qualities have given Galfenol a wide application in hydro-acoustics, precise control and vibration control systems. However, a little work on the model of magnetostriction for Galfenol, with considering the effects of anisotropy and dynamic losses, has been found. Therefore, it is interesting to investigate the model of magnetostriction and magnetization for Galfenol.

## II. CONSTITUTIVE THEORY

### A. An hysteretic Magnetostriction

In a hypothetical perfect magnetostrictive material, the magnetization is in accordance with the distribution of Boltzmann if the internal interaction among the domains is ignored. Based on the theory of free energy [1], magnetostriction,  $\lambda$ , with considering the effect of anisotropy, can be expressed as

$$\lambda(\sigma, H, T) = \iint \left( \frac{3}{2} \lambda_{100} (\alpha_1^2 \beta_{1R}^2 + \alpha_2^2 \beta_{2R}^2 + \alpha_3^2 \beta_{3R}^2 - \frac{1}{3}) + 3 \lambda_{111} (\alpha_1 \alpha_2 \beta_{1R} \beta_{2R} + \alpha_2 \alpha_3 \beta_{2R} \beta_{3R} + \alpha_1 \alpha_3 \beta_{1R} \beta_{3R}) \right) \cdot f(\theta, \varphi) \sin \theta d\theta d\varphi \quad (1)$$

where  $(\alpha_1, \alpha_2, \alpha_3)$ , and  $(\beta_{1R}, \beta_{2R}, \beta_{3R})$  are the direction cosine of magnetization and measuring direction along the magnetic crystal axis, respectively.  $\lambda_{100}$  and  $\lambda_{111}$  are magnetostriction constants in [100] and [111] direction, respectively.

The distribution probability function,  $f(\varphi, \theta)$ , which is the probability of magnetic crystal magnetization along the direction of  $(\varphi, \theta)$ , can be expressed as

$$f(\varphi, \theta) = \exp(-E/\Omega) / \iint \exp(-E/\Omega) \sin \theta d\theta d\varphi \quad (2)$$

where  $E$  is total free energy.  $\Omega$  is distributed parameter.

The total free energy can be written as

$$E = E_{ma} + E_H + E_{me} \quad (3)$$

where  $E_{ma}$  is anisotropy energy.  $E_H$  is magnetic field energy.  $E_{me}$  is stress-induced energy. The energy equations can be

acquired from (4)-(6).

$$E_{ma} = K_1 (\alpha_1^2 \alpha_2^2 + \alpha_1^2 \alpha_3^2 + \alpha_2^2 \alpha_3^2) + K_2 \alpha_1^2 \alpha_2^2 \alpha_3^2 \quad (4)$$

$$E_H = -\mu_0 M_s H (\alpha_1 \gamma_1 + \alpha_2 \gamma_2 + \alpha_3 \gamma_3) \quad (5)$$

$$E_{me} = -\frac{3}{2} \lambda_{100} \sigma (\alpha_1^2 \beta_1^2 + \alpha_2^2 \beta_2^2 + \alpha_3^2 \beta_3^2) - 3 \lambda_{111} \sigma (\alpha_1 \alpha_2 \beta_1 \beta_2 + \alpha_2 \alpha_3 \beta_2 \beta_3 + \alpha_1 \alpha_3 \beta_1 \beta_3) \quad (6)$$

where,  $K_1$  and  $K_2$  are constants of the first order and second order anisotropy.  $(\gamma_1, \gamma_2, \gamma_3)$  and  $(\beta_1, \beta_2, \beta_3)$  are the direction cosine of applied magnetic field and stress, respectively.  $M$  is magnetization,  $M_s$  is saturation magnetization,  $\sigma$  is stress, and  $\mu_0$  is free space permeability.

### B. Magnetostriction with Dynamic Losses

In the perfect magnetostrictive material, the domain wall translation events responsible for observable changes in magnetostriction and magnetization would occur completely reversibly. However, there are dynamic losses in the magnetization and magnetostrictive process of actual crystals, including eddy current loss  $E_{EC}$ , pinning loss  $E_{pin}$  and anomalous loss  $E_A$ . The total energy losses  $E_{loss}$  should be considered, and the energy loss equations can be written as [2]-[3]

$$E_{loss} = E_{EC} + E_{pin} + E_A \quad (7)$$

$$E_{EC} = \int \left( (\mu_0^2 d^2) / (2\rho\beta) \right) \cdot (dM/dt)^2 dt \quad (8)$$

$$E_{pin} = \mu_0 \int k dM \quad (9)$$

$$E_A = \int \left( (GdwH_0) / \rho \right)^{\frac{1}{2}} \left( (\mu_0 dM) / dt \right)^{\frac{3}{2}} dt \quad (10)$$

where  $\rho$  is resistivity, and  $k$  is pinning parameter. Those  $d$ ,  $G$ ,  $w$  and  $\beta$  are the structural parameters of Galfenol rod.  $H_0$  is a parameter representing the internal potential experienced by domain walls.

In order to incorporate the dynamic losses into the magnetostriction model, the energy losses,  $E_{loss}$ , is introduced to the total free energy equation (3), as shown in (11).

$$E = E_{ma} + E_H + E_{me} - E_{loss} \quad (11)$$

Therefore, the dynamic losses are taken into account when

calculating the distribution probability function,  $f(\varphi, \theta)$ , and magnetostriction,  $\lambda$ .

### C. Magnetization Model

The derivative of magnetization,  $dM/dt$ , can be expressed as

$$dM/dt = (dM/dH) \cdot (dH/dt) \quad (12)$$

Based on Jiles-Atherton model [4], the differential relation between magnetization and magnetic field can be expressed as

$$dM/dH = (M_{an} - M) / (\delta k - \tilde{\alpha}(M_{an} - M)) \quad (13)$$

where  $\tilde{\alpha} = \alpha + (9\lambda_s \sigma_0) / (2\mu_0 M_s^2)$ ,  $M_{an}$  is anhysteretic magnetization.  $\alpha$  is Weiss molecular field coefficient.  $\lambda_s$  is the saturation magnetostriction.  $\sigma_0$  is pre-stress. When the magnetic field increases, the parameter  $\delta$  is +1, and on the contrary  $\delta$  is -1.

The anhysteretic magnetization is calculated by effective field,  $H_e$ , and anisotropic field,  $H_F$ , as shown in (14)-(15).

$$M_{an} = M_s f(H_e + H_F) \quad (14)$$

$$f(H_e + H_F) = (\beta \cdot (H_e + H_F)) / (1 + \beta \cdot |H_e + H_F|) \quad (15)$$

There  $\beta$  is thermal agitation constant.

The effective field,  $H_e$ , is the sum of magnetic field,  $H$ , Weiss molecular field,  $\alpha M$ , and stress-induced magnetic field,  $H_\sigma$ , then,

$$H_e = H + \alpha M + H_\sigma \quad (16)$$

$$H_\sigma = (3/2) \cdot (\sigma / \mu_0) \cdot (d\lambda / dM) \quad (17)$$

According to Stoner-Wohlfarth model, the anisotropic field,  $H_F$ , can be obtained through the anisotropy energy,  $E_{ma}$  [5]-[7]. in the simple case of uniaxial crystalline anisotropy, the anisotropy energy is

$$E_{ma} = K_u |e_u \times M_{an}|^2 = K_u \cdot \cos^2 \phi \quad (18)$$

Here  $K_u$  is uniaxial anisotropic factor, and  $e_u$  is the unit vector in the direction of easy magnetization.  $\phi$  is the angle between the the anhysteretic magnetization and its easy axis direction.

Through the torque,  $\tau_u$ , exerted on magnetic dipoles due to anisotropy, the anisotropic field can be calculated by (19).

$$\begin{cases} \tau_u = dE_{ma} / d\phi \\ \tau_u = |M_{an} \times H_F| = M_s \cdot |H_F| \cdot \sin \phi \end{cases} \quad (19)$$

$$\Rightarrow H_F = ((2 \cdot K_u) / M_s^2) \cdot (M \cdot e_u) \cdot e_u$$

By (13)-(19), the value of the derivative of magnetization,  $dM/dt$ , can be obtained. Combined with (2)-(11), the magnetostriction can be calculated.

The magnetostriction model and magnetization model constitute the constitutive model of Galfenol rods. The hysteresis ( $M$ - $H$ ) and output characteristic ( $\lambda$ - $H$ ) under different magnetic fields and stresses can be calculated. The model can simultaneously describe the effects of anisotropy, stress and dynamic losses for Galfenol rods.

### III. CALCULATED AND EXPERIMENTAL RESULTS

Fig.1 displays the calculated curve (the black solid line) of magnetostriction versus magnetic field under 5 MPa pre-stress. The measurement of magnetostriction for Galfenol rod

specimen with 10 mm in diameter and 40 mm in length is made by WK-22 magnetic parameter measurement system. Using this equipment, the magnetic properties can be measured. The experimental result is also shown in Fig.1. From Fig.1, the magnetostriction quickly increases in low magnetic fields and saturates at the magnetic field of 2.5 kA/m. The calculated results are consistent with experimental ones when the magnetic field is less than 2.5 kA/m.

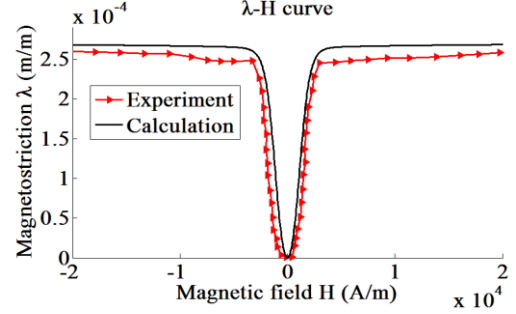


Fig. 1. Magnetic field dependence of magnetostriction for Galfenol rod

The magnetic field dependence of magnetization for Galfenol rod under 5 MPa pre-stress and 5 Hz frequency is shown in Fig.2. The magnetization increases rapidly in low magnetic fields and does slowly in the range of 8-16 kA/m. When the magnetic field is larger than 16 kA/m, the magnetization gradually reaches saturation. It is found that the calculated and experimental results match very well in low magnetic fields. It means that the proposed model can effectively describe the effects of anisotropy, stress and dynamic losses on the magnetostriction and magnetization of Galfenol rods, especially, in low magnetic fields.

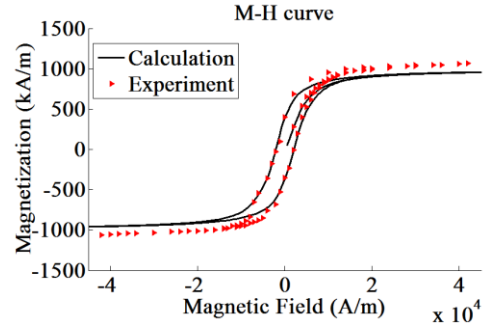


Fig. 2. Magnetic field dependence of magnetization for Galfenol rod

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